CSE330 Assignment 03

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Answer to question no 01.

a) Augmented matrix and first elimination step.

The linear step is :

$$4x_1 + 2x_2 + x_3 = 11$$
$$x_1 + 3x_2 + 2x_3 = 10$$
$$2x_1 + x_2 + 5x_3 = 13.$$

The augmented matrix [A|b] is :

$$\left[\begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 1 & 3 & 2 & 10 \\ 2 & 1 & 5 & 13 \end{array}\right]$$

First elimination step (Eliminate x_1 from R_2 and R_3):

For Row 2 : $m_{21} = 1/4 = 0.25$. Operation $R_2 \leftarrow R_2 - 0.25R_1$

$$\left[\begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 0 & 2.5 & 1.75 & 7.25 \\ 2 & 1 & 5 & 13 \end{array}\right]$$

For Row 3 : $M_{31} = 2/4 = 0.5$. Operation $R_3 \leftarrow R_3 - 0.5R_1$

$$\left[\begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 0 & 2.5 & 1.75 & 7.25 \\ 0 & 0 & 4.5 & 7.5 \end{array}\right]$$

b) Upper triangular form and multipliers.

Since the element a_{32} is already 0, the matrix is already in upper triangular form. The multiplier m_{32} is :

$$m_{32} = 0/2.5 = 0$$

Final upper triangular matrix u :

$$u = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix}$$

c) Backward Substitution.

1. from R_3 : $4.5x_3 = 7.5$

$$\Rightarrow x_3 = \frac{7.5}{4.5} = 1.6666... \approx 1.667$$

2. From R_2 : $2.5x_2 + 1.75(1.6666...) = 7.25$

$$\Rightarrow 2.5x_2 = 7.25 - 2.9166... = 4.3333..$$

$$\Rightarrow x_2 = 4.3333/2.5 = 1.7333$$

3. From R_1 : $4x_1 + 2(1.7333...) + 1.6666... = 11.$

$$\Rightarrow 4x_1 = 11 - 3.4666... - 1.6666... = 5.8666$$

$$\Rightarrow x_1 = \frac{5.8666}{4} = 1.467.$$

$$x = \begin{bmatrix} 1.467 \\ 1.733 \\ 1.667 \end{bmatrix}$$

Answer to question no. 02

LU Decomposition method.

a) Decomposition into LU.

From the gaussian elimination steps in Q.1, we identified the multipliers:

$m_{21} = 0.25$

$m_{31} = 0.5$

$m_{32} = 0$

Lower triangular Matrix L (with unit diagonal), $L = \begin{bmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$

Upper triangular matrix U: $\begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix}$

b) $Ly = b$ [forward Substitution]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 11 \\ 10 \\ 13 \end{bmatrix}$$

$\therefore y_1 = 11$

$\Rightarrow 0.25(11) + y_2 = 10$

$\therefore y_2 = 7.25$

$\Rightarrow 0.5(11) + 0(y_2) + y_3 = 13$

$\therefore y_3 = 7.5.$

Hence, Intermediate vector

$y = [11, 7.25, 7.5]^T.$

c) Solve $Ux = y$ (Backward Substitution)

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 11 \\ 7.25 \\ 7.5 \end{bmatrix}$$

Solving this yield the same results as question 1(c):

$x_3 = 1.667$ $x_2 = 1.733$, $x_1 = 1.467$.