CSE-330

Assignment-2

Name : Khandkar Sajiduzzaman

ID : 24241216

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Ans to Ques No - 1

a) $g(x) = \frac{6 + x^3 - 2x^2}{5} = \frac{1}{5}(6 + x^3 - 2x^2)$

$\lambda = |g'(x_*)|$

$f(x) = x^3 - 2x^2 - 5x + 6$

$x_* = -2, 3, 1$

$g'(x) = \frac{1}{5}(6 + x^3 - 2x^2) d/dx$

$= \frac{-4x + 3x^2}{5}$

$\lambda = |g'(-2)| = 4 \longrightarrow$ divergent

$\lambda = |g'(3)| = 3 \longrightarrow$ divergent

$\lambda = |g'(1)| = 0.2 \longrightarrow$ linearly Convergent

It converges only near $x_* = 1$

and it is linearly Convergent, $\lambda = 0.2$

b)

$x_0 = 0.5 \quad x_{k+1} = g(x_k)$

$$x_1 = g(x_0) = \frac{6 + (0.5)^3 - 2 \times (0.5)^2}{5}$$

$$x_1 = 1.125$$

$$x_2 = g(x_1)$$
$$= \frac{6 + (1.125)^3 - 2 \times (1.125)^2}{5}$$
$$= 0.9785$$

$$x_3 = g(x_2) = \frac{6 + (0.9785)^3 - 2 \times (0.9785)^2}{5}$$
$$= 1.0043$$

$$x_4 = g(x_3) = \frac{6 + (1.0043)^3 - 2 \times (1.0043)^2}{5}$$
$$= 0.9991$$

$$x_5 = g(x_4) = \frac{6 + (0.9991)^3 - 2 \times (0.9991)^2}{5}$$
$$= 1.0001$$

$$x_6 = g(x_5) = \frac{6 + (1.0001)^3 - 2 \times (1.0001)^2}{5}$$
$$= 0.9999$$

$$x_7 = \frac{6 + (0.9999)^3 - 2\times(0.9999)^2}{5}$$

$$= 1.0000$$

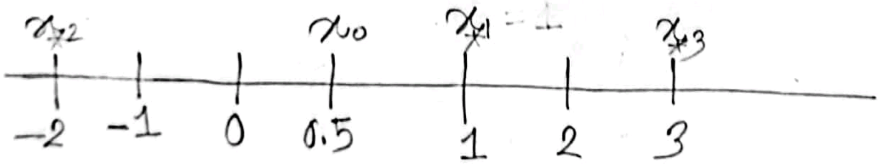
$$x_8 = \frac{6 + (1.0000)^3 - 2\times(1.0000)^2}{5}$$

$$= 1$$

$$\therefore x_8 = x_* = g(x_7) = 1$$

it converges to $x_* = 1$ root because of the choice of the starting point. Accorging to Contraction Mapping Theorem $x_{K+1} = g(x_K)$ converges to the x_* that is close to the x_0 point

Here,



Here $x_* = 1$ is the closest point to x_0.

Ans to Ques No-2

a)

$$f'(x) = \frac{rise}{run} = \frac{f(x_k) - 0}{x_k - x_{k+1}}$$

$$x_k - x_{k+1} = \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$g(x_k) = x_k - \frac{f(x_k)}{f'(x_k)}$$

b)

for super linear convergence,

$$\lambda = |g'(x_*)| = 0$$

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$g'(x) = 1 - \frac{f'(x)f'(x) - f(x)f''(x)}{(f'(x))^2}$$

at any x_*, $f(x_*) = 0$

So, $$g'(x_*) = 1 - \frac{f'(x_*)f'(x_*) - f(x_*)f''(x_*)}{(f'(x_*))^2}$$

$$= 1 - \frac{(f'(x_*))^2}{(f'(x_*))^2}$$

$$= 1 - 1$$

$$= 0$$

$$\therefore g'(x_*) = 0$$

$$\therefore \eta = |g'(x_*)| = 0$$

Ans to Question No - 3

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} + \frac{f''(\xi)}{2!}(x_k - x_{k-1})$$

$$f'(x_k) \approx \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

iteration formula fore the Secant Method,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{f(x_k)}{\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}}$$

$$x_{k+1} = x_k - f(x_k)\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$$

Here, it approximates the derivative in Newton's method by ignoring the truncation error from Numerical differentiation.

b) $f(x) = x^3 - 2x^2 - 5x + 6$ $\qquad x_{k+1} = x_k - f(x_k)\dfrac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

K	x_{k-1}	x_k	x_{k+1}
1	0.00000	0.50000	1.04347
2	0.50000	1.04347	1.00189
3	1.04347	1.00189	0.99998
4	1.00189	0.99998	1.00000