Assignment-1

CSE-330

Name: MD. Tajwar Islam

ID: 23201280

Section: 12

Question-1

Solution:

Given, $x = (3.165)_{10}$

(a) Integer,

$2 \mid 3$ — 1

$2 \mid 1$ — 1

0

Floating point,

$0.165 \times 2 = 0.33$ — 0

$0.33 \times 2 = 0.66$ — 0

$0.66 \times 2 = 1.32$ — 1

$0.32 \times 2 = 0.64$ — 0

$0.64 \times 2 = 1.28$ — 1

$0.28 \times 2 = 0.56$ — 0

$0.56 \times 2 = 1.12$ — 1

$0.12 \times 2 = 0.24$ — 0

$0.24 \times 2 = 0.48$ — 0

So, binary representation is $(11.001010100)_2$ (Ans)

(b) Standared form,

$$(0.11001010100)_2 \times 2^2$$

Now, for $m = 4$,

$$(0.1100)_2 \times 2^2$$

and $m = 6$,

$$(0.110010)_2 \times 2^2$$

(c) Decimal of $(0.1100)_2 \times 2^2$:

$$\left(1\times 2^{-1} + 1\times 2^{-2} + 0 + 0\right) \times 2^2$$

$$= (3)_{10}$$

Decimal of $(0.110010)_2 \times 2^2$:

$$\left(1\times 2^{-1} + 1\times 2^{-2} + 0 + 0 + 1\times 2^{-5} + 0\right) \times 2^2$$

$$= \left(\frac{25}{8}\right)_{10} = (3.125)_{10}$$

Now, rounding error for $m = 4$,

$$(3.165 - 3)_{10} = (0.165)_{10}$$

and for $m = 6$,

$$(3.165 - 3.125)_{10} = (0.04)_{10}$$

Question-2

Solution:

Given,

$$v(t) = -\frac{3}{4}t^2 + \frac{19}{2}t - 6 \; ; t_0 = 2 \; ; t_1 = 4 \; ; t_2 = 6$$

(a)

t	$v(t)$
2	10
4	20
6	24

Here, $P_2(t) = a_0 + a_1 t + a_2 t^2$

and the system,

$$a_0 + a_1 2 + a_2 2^2 = 10$$

$$a_0 + a_1 4 + a_2 4^2 = 20$$

$$a_0 + a_1 6 + a_2 6^2 = 24$$

By Vandermonde Matrix,

$$\begin{bmatrix} 1 & 2 & 2^2 \\ 1 & 4 & 4^2 \\ 1 & 6 & 6^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 24 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -1.25 & 2 & -0.75 \\ 0.125 & -0.25 & 0.125 \end{bmatrix}$$

So,

Solving the equation,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 1 \\ -1.25 & 2 & -0.75 \\ 0.125 & -0.25 & 0.125 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 24 \end{bmatrix}$$

We get,

$$a_0 = -6 \; ; \; a_1 = \frac{19}{2} \; ; \; a_2 = -\frac{3}{4}$$

So, $P(t) = -6 + \frac{19}{2}t - \frac{3}{4}t^2$ (Ans)

Now,

acceleration at $t = 7$ seconds,

~~$P(7) = -6 + \frac{19}{2}(7) - \frac{3}{4}(7)^2$~~

$$a(t) = v'(t) = \frac{d}{dt}\left[-\frac{3}{4}t^2 + \frac{19}{2}t - 6\right]$$

$$= -\frac{3}{2}t + \frac{19}{2}$$

$$\therefore a(7) = -\frac{3}{2}(7) + \frac{19}{2}$$

$$= -1 \quad \text{(Ans)}$$

(b)

t	v(t)
2	10
4	20
6	24

For lagrange,

$$P(t) = \sum_{i=0}^{n} v(t_i) L_i(t)$$

Now,

$$L_0(t) = \frac{t-4}{2-4} \times \frac{t-6}{2-6} = \frac{(t-4)(t-6)}{12}$$

$$L_1(t) = \frac{t-2}{4-2} \times \frac{t-6}{4-6} = \frac{(t-2)(t-6)}{-4}$$

$$L_2(t) = \frac{t-2}{6-2} \times \frac{t-4}{6-4} = \frac{(t-2)(t-4)}{8}$$

So,

$$P(t) = v(t_0) L_0(t) + v(t_1) L_1(t) + v(t_2) L_2(t)$$

$$= \frac{10(t-4)(t-6)}{12} + \frac{20(t-2)(t-6)}{-4} + \frac{24(t-2)(t-4)}{8}$$

$$= \frac{5}{6}(t-4)(t-6) - 5(t-2)(t-6) + 3(t-2)(t-4)$$

(Ans)